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On supergravity solutions of space-like Dp-branes

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Abstract: Recently the time dependent solutions of type II supergravities in $d = 10$, with the metric having the symmetry $\mathrm{ISO}(p+1) \times \mathrm{SO}(8-p, 1)$ have been given by two groups (Chen-Gal'tsov-Gutperle (CGG), [hep-th/0204071] and Kruczenski-Myers-Peet (KMP), [hep-th/0204144]). The supergravity solutions correspond to space-like Dp-branes in type II string theory. While the CGG solution is a four parameter solution, the KMP solution is a three parameter solution and so in general they are different. This difference can be attributed to the fact that unlike the CGG solution, KMP uses a specific boundary condition for the metric and the dilaton field. It is shown that when we impose the boundary conditions used in the KMP solution to the CGG solution then both become three parameter solutions and they map to each other under a coordinate transformation along with a Hodge duality of the field strength. We also give the relations between the parameters characterizing the two solutions.

Keywords: Superstrings and Heterotic Strings, D-branes, Superstring Vacua, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Recently there has been a lot of interest in constructing and understanding the time dependent solutions in string/M theory. The major motivations for studying these solutions are: (a) they might provide the stringy resolution of space-like cosmological singularity behind the black hole horizon, (b) they might provide a concrete realization of dS/CFT correspondence [1, 2] in string theory. The issue of singularity has been addressed in the context of time dependent orbifold model in string theory in [3]–[7]. Also many physical issues like observables, perturbation theory, particle creation have been addressed in a simple time dependent string background in [8, 9].

Space-like p-branes (Sp-branes) are topological defects localized in $(p+1)$ dimensional space-like surfaces and are known to arise in string/M theory (also in some field theories) as time dependent solutions [10]. Space-like Dp-branes (SDp-branes) arise when the time-like as well as $(8-p)$ space-like coordinates of the open string satisfy Dirichlet boundary condition [11, 12] and carry the same kind of RR charge as the ordinary (time-like) Dp-branes. They can also be understood [13] to arise from unstable D$(p+1)$-brane or ordinary Dp-brane-anti-Dp-brane pair as the time-like tachyonic kink solution. The supergravity description of the SDp-branes are particularly interesting to understand the time-like holography principle of the dS/CFT correspondence [14].

The supergravity description of SDp-branes has been given by Chen-Gal'tsov-Gutperle (CGG) [15] and also by Kruczenski-Myers-Peet (KMP) [16]. These two solutions look quite different and in fact the methods used to obtain these two solutions are also completely different. In the first case CGG started with a coupled dilaton, gravity and a q-form field strength system in d space-time dimensions which is the bosonic sector of low energy effective action of various string theories or M theory compactified to d dimensions. The non-linear differential equations resulting from the effective action are then solved with an ansatz for the metric to have the symmetry structure $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$. The resulting time dependent solution for $d=10$ and the dilaton coupling $a=(p-3)/2$ represents the localized (we take $k=q$ in the CGG solution and for $d=10$, $q=8-p$)

SDp-branes of type II string theory [15]. These are magnetically charged SDp-branes in the Einstein-frame metric and are dependent on four parameters. On the other hand, KMP started with the eleven dimensional solution of the equations of motion of pure Einstein gravity with appropriate symmetries [21]. Then they performed a rotation mixing the eleventh dimension and one of the space-like dimensions. The dimensional reduction of the eleventh dimension produces the SD0-brane solution of type IIA string theory smeared in some number of transverse directions. The usual solution generating technique of T-duality [22, 23] in the transverse directions then gives the required electric SDp-brane solutions. Demanding isotropy in $(p+1)$ directions gives a three parameter solution of SDp-branes in the string frame [16] (see also [17, 18, 19, 20] for some earlier work).

Since the CGG solution is a four parameter solution, whereas the KMP solution is a three parameter solution, they are in general different. The purpose of this paper is to show under what condition they will be the same as both of them represent the SDp-brane supergravity solutions. In fact we will point out that unlike the CGG solution, the KMP solution uses specific boundary conditions for the metric as well as for the dilaton. We show that when the same boundary conditions used by the KMP solution is imposed upon the CGG solution, then the two solutions indeed map to each other[1] under a coordinate transformation along with a Hodge duality of the field strength (since in the CGG case SDp-branes are magnetically charged whereas in the KMP case they are electrically charged). The coordinate transformation has the form:

$$\begin{aligned} \frac{t}{\omega} &= \left[\tanh\frac{(7-p)\hat{t}}{2}\right]^{-\frac{1}{7-p}} \\ \text{or,}\quad \hat{t} &= -\frac{1}{7-p}\ln\left[\frac{1-(\frac{\omega}{t})^{7-p}}{1+(\frac{\omega}{t})^{7-p}}\right] \end{aligned} \tag{1.1}$$

where t is the time-like coordinate in the KMP solution and ω is a parameter and $\hat{t}$ is the time-like coordinate in CGG. Note that as $t \to \infty$, $\hat{t} \to 0$ and as $t \to \omega$, $\hat{t} \to \infty$. We will also give the relations between the parameters characterizing the two solutions. If we demand that the string-frame metric becomes flat (in Rindler coordinates) and $e^{2\phi}$ approaches unity for $t \to \infty$ (or $\hat{t} \to 0$), to the CGG solution (as has been used by the KMP solution), then both the solutions become three parameter solution and they map to each other under the above coordinate transformation along with a Hodge duality of the field strength. In fact, when we use these boundary conditions then one of the parameters in the CGG solution gets related to the other three paramaters in a specific way. This is the reason that in this case both the solutions become three parameter solutions which is necessary for the complete mapping of these two solutions. Thus we show that under these circumstances the CGG solution and the KMP solution become identical to each other.

The organization of this paper is as follows. In section 2, we compare the space-like M5-brane solutions obtained by Gutperle-Strominger (GS) and KMP and fix our notations. In section 3, we discuss the mapping of SDp-brane solutions of CGG and KMP. We summarize our conclusion in section 4.

[1] A similar mapping for the static solution has been pointed out in [15] by comparing their solution with that of [24].

2. SM5-brane solutions

In this section we will discuss the equivalence of SM5-brane solutions obtained by GS [10] and KMP [16]. We will point out that in both cases the SM5-brane solutions are two-parameter solutions and they map to each other under a coordinate transformation as well as a Hodge duality of the field strength. In [10] the SM5-brane solution is obtained by solving the equation of motion resulting from the bosonic action of $d = 11$ supergravity. By imposing the appropriate symmetry the supergravity solution of SM5-brane is found to have the form,

$$\begin{aligned} ds^2 &= -e^{2A}d\hat{t}^2 + e^{2C}dH^2_{d-p-2} + e^{2B}dx^2_{(p+1)} \\ *F_{p+2} &= e^{2B(p+1)}b\; d\hat{t}\wedge dx_1 \wedge \cdots \wedge dx_{p+1} \end{aligned} \tag{2.1}$$

where A, B, C are functions of $\hat{t}$ only and satisfy the gauge condition

$$-A + (p+1)B + (d-p-2)C = 0 \tag{2.2}$$

We have written the solution in (2.1) such that it can be generalized for SDp-branes to be discussed in the next section. In this particular case of SM5-brane solution $p = 5$ and $d = 11$. dH^2_{d-p-2} is the line element for the $(d-p-2)$-dimensional hyperbolic space with negative curvature and $dx^2_{(p+1)}$ is the same for the flat $(p+1)$-dimensional euclidean space.

Since the functions A, B, C satisfy the gauge condition(2.2), they can be expressed in terms of two functions as follows,

$$A = (d-p-2)g(\hat{t}) - \frac{(p+1)}{(d-p-3)}f(\hat{t})\,, \qquad B = f(\hat{t})\,, \qquad C = g(\hat{t}) - \frac{(p+1)}{(d-p-3)}f(\hat{t}) \tag{2.3}$$

We will use these forms for the SDp-brane solution of type II string theory in the next section for $d = 10$. Solving the equations of motion the functions g and f are found to have the forms[2]

$$\begin{aligned} f(\hat{t}) &= \frac{2}{\chi}\ln\frac{\alpha}{\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)} + \frac{1}{\chi}\ln\frac{(d-2)\chi}{(d-p-3)b^2} \\ g(\hat{t}) &= \frac{1}{d-p-3}\ln\frac{\beta}{\sinh(d-p-3)\beta(\hat{t}-t_1)} \end{aligned} \tag{2.4}$$

where $\chi = 12$ in this case and α, β are integration constants satisfying

$$\frac{(d-2)\chi\alpha^2}{2(d-p-3)} - (d-p-2)(d-p-3)\beta^2 = 0 \tag{2.5}$$

Also, t_0 and t_1 are two other integration constants. Note that t_1 can be absorbed by shifting $\hat{t}$ coordinate. For SM5 solution (2.5) reduces to

$$3\alpha^2 - 2\beta^2 = 0 \tag{2.6}$$

[2]These are actually solutions of d-dimensional gravity coupled to $(d-p-2)$-form field strength system.

and the functions $f(\hat{t})$ and $g(\hat{t})$ simplify to [10]

$$f(\hat{t}) = \frac{1}{6}\ln\frac{\beta}{\cosh\sqrt{24}\beta(\hat{t}-t_0)} - \frac{1}{12}\ln\left(\frac{b^2}{24}\right) \tag{2.7}$$

$$g(\hat{t}) = \frac{1}{3}\ln\frac{\beta}{\sinh 3\beta\hat{t}} \tag{2.8}$$

where we have eliminated the integration constant α using (2.6). Furthermore, the constant β can also be eliminated by scaling $\hat{t} \to \hat{t}/\beta$ and $x_i \to x_i/\beta^{1/6}$ for $i = 1, \ldots, 6$. We remark here that β can not be eliminated by similar rescaling of coordinates only for SDp solutions to be discussed later. Thus eliminating β for this case the metric and the 7-form dual field strength take the following forms,

$$\begin{aligned} ds^2 &= \left(\frac{b^2}{24}\right)^{1/3} \frac{\left(\cosh\sqrt{24}(\hat{t}-t_0)\right)^{2/3}}{(\sinh 3\hat{t})^{8/3}} \left(-d\hat{t}^2 + \sinh^2 3\hat{t} dH_4^2\right) \\ &\quad + \left(\frac{b^2}{24}\right)^{-1/6} \frac{1}{(\cosh\sqrt{24}(\hat{t}-t_0))^{1/3}} dx_{(6)}^2 \\ *F_7 &= e^{12f} b\, d\hat{t} \wedge dx_1 \wedge \cdots \wedge dx_6 \end{aligned} \tag{2.9}$$

The above equation represents the SM5-brane solution characterized by two parameters b and t_0. We now make a coordinate transformation

$$\hat{t} = -\frac{1}{3}\ln\frac{f_-}{f_+} \tag{2.10}$$

where

$$f_\pm = 1 \pm \left(\frac{\omega}{t}\right)^3 \tag{2.11}$$

Then we find,

$$\frac{1}{(\sinh 3\hat{t})^{8/3}}\left(-d\hat{t}^2 + \sinh^2 3\hat{t} dH_4^2\right) = \frac{(f_+f_-)^{2/3}}{2^{2/3}\omega^2}(-dt^2 + t^2 dH_4^2) \tag{2.12}$$

and we rewrite

$$\begin{aligned} \left(\frac{b^2}{24}\right)^{1/2} \cosh\sqrt{24}(\hat{t}-t_0) &= 2\omega^3\left[\cos^2\theta\left(\frac{f_-}{f_+}\right)^{-\sqrt{8/3}} + \sin^2\theta\left(\frac{f_-}{f_+}\right)^{\sqrt{8/3}}\right] \\ &= 2\omega^3 F \end{aligned} \tag{2.13}$$

where we have defined

$$\begin{aligned} 2\omega^3\cos^2\theta &= \frac{1}{2}e^{-\sqrt{24}t_0}\left(\frac{b^2}{24}\right)^{1/2} \\ 2\omega^3\sin^2\theta &= \frac{1}{2}e^{\sqrt{24}t_0}\left(\frac{b^2}{24}\right)^{1/2} \end{aligned} \tag{2.14}$$

Note here that θ is the mixing angle of the eleventh dimension and one of the space-like dimensions used in [16] to construct the SM5-brane solution. Now using (2.12)–(2.14), the metric and the field strength in (2.9) can be rewritten as,

$$\begin{aligned} ds^2 &= F^{2/3}\left(f_+ f_-\right)^{2/3}\left[-dt^2 + t^2 dH_4^2\right] + F^{-1/3} dx_{(6)}^2 \\ F_4 &= -b\,\epsilon(H_4) = -6\sin\theta\cos\theta\left(2\sqrt{\frac{8}{3}}\right)\omega^3\epsilon(H_4) \end{aligned} \tag{2.15}$$

This is precisely the same form of the metric and the field strength obtained in [16] for the SM5-brane solution. Note that the coordinates x_i for $i = 1, 2, \ldots, 6$ are rescaled while we write (2.15) by $x_i \to (2\omega^3)^{1/6} x_i$. Also we have taken the Hodge dual of the field strength in (2.9) to write the first expression of F_4 in (2.15). The second expression is written using (2.14). $\epsilon(H_4)$ represents the volume form of 4-dimensional hyperbolic space. So both the GS solution and the KMP solution are two parameter solutions. The relations between the GS parameters (t_0, b) and the KMP parameters (ω, θ) follow from (2.14) as,

$$\begin{aligned} \omega^3 &= \frac{1}{2}\left(\frac{b^2}{24}\right)^{1/2}\cosh\sqrt{24}t_0 \qquad \Rightarrow \qquad b = 4\sqrt{24}\omega^3\sin\theta\cos\theta \\ \tan\theta &= e^{\sqrt{24}t_0} \qquad \Rightarrow \qquad t_0 = \frac{1}{\sqrt{24}}\ln(\tan\theta) \end{aligned} \tag{2.16}$$

Thus we have shown the exact mapping of the two apparently different looking solutions of SM5-brane obtained in [10] and [16] by the coordinate transformation (2.10).

3. SDp-brane solutions

In this section we show the equivalence of the SDp-brane solutions obtained in [15] and [16] along the same line as in the previous section. We point out that the CGG solution is a four parameter solution and the KMP solution is a three parameter solution. The difference is because the KMP solution uses a specific boundary condition that the string-metric in their solution becomes flat in Rindler coordinates and $e^{2\phi}$ approaches unity as $t \to \infty$ (or, $\hat{t} \to 0$), whereas for the CGG solution they remain arbitrary. This arbitrariness shows up as an additional parameter in the CGG solution. However, as we impose these additional restrictions in the CGG solution, we find that one of the parameters in the CGG solution is removed and then these two solutions map to each other. Let us start with the CGG solution which has the form as given in (2.1) along with the gauge condition (2.2) where now $d = 10$ and there is a dilaton $\phi(\hat{t})$. Solving the equations of motion the functions $f(\hat{t})$, $g(\hat{t})$ and $\phi(\hat{t})$ were obtained in [15] to have the forms,

$$\begin{aligned} f(\hat{t}) &= \frac{2}{\chi}\ln\frac{\alpha}{\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)} + \frac{1}{\chi}\ln\frac{8\chi}{(7-p)b^2} - \frac{(p-3)c_1}{2\chi}\hat{t} - \frac{(p-3)c_2}{2\chi} \\ g(\hat{t}) &= \frac{1}{7-p}\ln\frac{\beta}{\sinh(7-p)\beta(\hat{t}-t_1)} \\ \phi(\hat{t}) &= \frac{4(p-3)}{(7-p)}f(\hat{t}) + c_1\hat{t} + c_2 \\ \text{and} \quad F_{8-p} &= b\epsilon(H_{8-p}) \end{aligned} \tag{3.1}$$

In the above $\chi = 32/(7-p)$ and α, β, t_0, t_1, c_1 and c_2 are the integration constants. b is related to the magnetic charge of the solution. We remark that c_1 and c_2 are two more integration constants (than in the previous case) which appeared while solving the dilaton equation of motion. The constants α, β and c_1 are related by,

$$\frac{(p+1)}{\chi}c_1^2 + \frac{4\chi}{(7-p)}\alpha^2 - (8-p)(7-p)\beta^2 = 0 \tag{3.2}$$

Note that we can absorb t_1 by shifting $\hat{t}$ and therefore the solution depends on six parameters β, t_0, c_1, c_2 and b with a relation between α, β and c_1 given in (3.2). Now in order to show the mapping we write the full CGG solution of SDp-branes using (3.1) and (2.1)–(2.3) as,

$$\begin{aligned}
ds^2 &= \left(\frac{\beta}{\sinh(7-p)\beta\hat{t}}\right)^{\frac{2(8-p)}{(7-p)}} \left(\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{1/2} \left(\frac{(7-p)b}{16\alpha}\right)^{1/2} e^{\frac{p+1}{8}(c_1\hat{t}+c_2)} \times \\
&\quad \times \left[-d\hat{t}^2 + \frac{\sinh^2(7-p)\beta\hat{t}}{\beta^2}dH_{8-p}^2\right] + \\
&\quad + \left(\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{-1/2} \left(\frac{(7-p)b}{16\alpha}\right)^{-1/2} e^{\frac{7-p}{8}(c_1\hat{t}+c_2)} dx_{(p+1)}^2 \\
e^{2\phi} &= \left(\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{\frac{3-p}{2}} \left(\frac{(7-p)b}{16\alpha}\right)^{\frac{3-p}{2}} e^{\frac{(p+1)(7-p)}{8}(c_1\hat{t}+c_2)} \\
F_{8-p} &= b\epsilon(H_{8-p})
\end{aligned} \tag{3.3}$$

Here we have written the metric in the string frame by multiplying the expression of (2.1) with $e^{\phi/2}$ since the KMP metric is given in the string frame. Also in order to compare with the KMP solution we have to dualize the field strength. Now we would like to point out that in the above solution β can not be eliminated by just rescaling the coordinate $\hat{t}$ and x_i, for $i = 1, \ldots, p+1$, as has been done for the SM5-brane in the previous section. However, we find that there is a unique way one can eliminate β and this is done by renaming the parameters as follows,

$$\begin{aligned}
\frac{\alpha}{\beta} &\to \alpha \\
\beta t_0 &\to t_0 \\
\frac{c_1}{\beta} &\to c_1 \\
\beta^{\frac{p-3}{2}} e^{\frac{(p+1)(7-p)}{8}c_2} &\to e^{\frac{(p+1)(7-p)}{8}c_2} \\
b &\to b
\end{aligned} \tag{3.4}$$

along with the coordinate rescaling $\hat{t} \to \hat{t}/\beta$ and $x_i \to x_i/\beta^{1/(p+1)}$, for $i = 1, \ldots, p+1$. Note that the renaming of the parameter c_2 is not necessary for $p = 3$. In fact, in this case the renaming of other parameters and the coordinate rescalings is enough to eliminate β completely from the solution (3.3). As $p = 3$ case is different from the other cases we will discuss the mapping for this case at the end of this section. Also, it should be emphasized that only when β is eliminated $e^{2\phi}$ approaching unity and the string metric becoming flat as $\hat{t} \to 0$ can be achieved. Now the solution depends on five parameters α, t_0, c_1, c_2, b with a relation between α and c_1 of the form (see eq. (3.2))

$$\frac{(p+1)}{\chi} c_1^2 + \frac{4\chi}{(7-p)} \alpha^2 = (8-p)(7-p) \tag{3.5}$$

Therefore if we eliminate one of c_1 and α, then the solution would depend on four parameters. Eliminating β the solution (3.3) reduces to,

$$\begin{aligned} ds^2 = & \left(\frac{1}{\sinh(7-p)\hat{t}}\right)^{\frac{2(8-p)}{(7-p)}} \left(\cosh \frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{1/2} \left(\frac{(7-p)b}{16\alpha}\right)^{1/2} e^{\frac{p+1}{8}(c_1\hat{t}+c_2)} \times \\ & \times \left[-d\hat{t}^2 + \sinh^2(7-p)\hat{t} dH_{8-p}^2\right] + \\ & + \left(\cosh \frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{-1/2} \left(\frac{(7-p)b}{16\alpha}\right)^{-1/2} e^{\frac{7-p}{8}(c_1\hat{t}+c_2)} dx_{(p+1)}^2 \end{aligned} \tag{3.6}$$

with the dilaton and the $(8-p)$-form retaining the same form as given in (3.3). So unless we impose any further condition the SDp-brane solution would depend on four parameters and will be different from the KMP solution. We will now try to map this four parameter solution to the KMP solution and see how the parameters in these two solutions are related.

In order to do this we make a coordinate transformation (1.1) i.e.

$$\hat{t} = -\frac{1}{7-p} \ln\left(\frac{f_-}{f_+}\right) \tag{3.7}$$

where $f_\pm$ are defined as,

$$f_\pm = 1 \pm \left(\frac{\omega}{t}\right)^{7-p} \tag{3.8}$$

Then we find,

$$\frac{1}{\left(\sinh(7-p)\hat{t}\right)^{\frac{2(8-p)}{(7-p)}}} \left[-d\hat{t}^2 + \sinh^2(7-p)\hat{t}\ dH_{8-p}^2\right] = \frac{(f_+f_-)^{\frac{2}{(7-p)}}}{2^{\frac{2}{(7-p)}}\omega^2} \left(-dt^2 + t^2 dH_{8-p}^2\right) \tag{3.9}$$

Also we rewrite

$$\begin{aligned} & \frac{(7-p)b}{16\alpha} e^{\frac{(p+1)}{4}(c_1\hat{t}+c_2)} \cosh \frac{\chi\alpha}{2}(\hat{t}-t_0) = \\ & \quad = 2^{\frac{4}{(7-p)}} \omega^4 \left(\frac{f_-}{f_+}\right)^{\frac{2n(p-1)}{(7-p)}} \left[\cos^2\theta \left(\frac{f_-}{f_+}\right)^{\frac{n-m}{2}} + \sin^2\theta \left(\frac{f_-}{f_+}\right)^{\frac{n+m}{2}}\right] \\ & \quad = 2^{\frac{4}{(7-p)}} \omega^4 \left(\frac{f_-}{f_+}\right)^{\frac{2n(p-1)}{(7-p)}} F \end{aligned} \tag{3.10}$$

where we have defined

$$\begin{aligned} \cos^2\theta &= \frac{1}{2} e^{-\frac{\chi\alpha}{2}t_0} \left(\frac{(7-p)b}{16\alpha}\right) e^{\frac{(p+1)(7-p)}{4(3-p)}c_2} \\ \sin^2\theta &= \frac{1}{2} e^{\frac{\chi\alpha}{2}t_0} \left(\frac{(7-p)b}{16\alpha}\right) e^{\frac{(p+1)(7-p)}{4(3-p)}c_2} \end{aligned} \tag{3.11}$$

We find that eq. (3.10) will be consistent if m, n and ω satisfy the following relations,

$$m = \frac{32\alpha}{(7-p)^2}\,, \qquad n = -\frac{c_1}{6}\,, \qquad \omega = \frac{1}{2^{\frac{1}{(7-p)}}} e^{\frac{(p+1)}{4(p-3)}c_2} \tag{3.12}$$

Now using (3.9) and (3.10) the metric (3.6) takes the form,

$$\begin{aligned} ds^2 &= F^{1/2}\left(\frac{f_-}{f_+}\right)^{\frac{n(p-1)}{(7-p)}} (f_+f_-)^{\frac{2}{7-p}} \left(-dt^2 + t^2 dH^2_{8-p}\right) \\ &\qquad\qquad + F^{-1/2}\left(\frac{f_-}{f_+}\right)^n dx^2_{(p+1)} \end{aligned} \tag{3.13}$$

This is precisely the form of the metric for the SDp-brane obtained in [16]. In writing (3.13) we have rescaled the coordinates x_i, for $i = 1, 2, \ldots, p+1$, by $x_i \to (2\omega^{7-p})^{1/(p+1)} x_i$. Also from (3.11) we find

$$\begin{aligned} \tan\theta &= e^{\frac{16\alpha t_0}{(7-p)}} \\ c_2 &= -\frac{4(3-p)}{(p+1)(7-p)} \ln\left(\frac{(7-p)b}{16\alpha}\cosh\frac{16\alpha t_0}{(7-p)}\right) \end{aligned} \tag{3.14}$$

We thus find that the parameter c_2 is determined in terms of α, b and t_0 and so, we are left with four parameter solution α, b, c_1, t_0 with a relation between c_1 and α given in (3.5), just like the KMP solution which is dependent on four parameters m, n, ω and θ with a relation between m and n [16]. It should be emphasized that unlike the other conditions given in (3.12) and (3.14), the second relation of (3.14) does not relate the CGG parameters with the KMP parameters. Rather it is a relation among the parameters of the CGG solution itself and this reduces the number of parameters from four to three. The reason behind this is while trying to map the CGG solution to the KMP solution we are imposing the boundary condition (i.e. the metric becomes flat and $e^{2\phi}$ approaches unity as $\hat{t} \to 0$) of the KMP solution here. Note that using (3.12) and (3.5) we get the relation between m and n as,

$$9(p+1)n^2 + (7-p)m^2 = 8(8-p) \tag{3.15}$$

which is the same as in ref. [16, eq.(15)]. The relations between the CGG parameters and the KMP parameters are

$$\begin{aligned} m &= \frac{32\alpha}{(7-p)^2} \\ n &= -\frac{c_1}{6} \\ \omega &= \left[\frac{(7-p)b}{32\alpha}\cosh\frac{16\alpha t_0}{(7-p)}\right]^{1/(7-p)} \\ \tan\theta &= e^{\frac{16\alpha t_0}{(7-p)}} \end{aligned} \tag{3.16}$$

or, inverting the relations we get,

$$\alpha = \frac{(7-p)^2 m}{32}$$

$$
\begin{aligned}
c_1 &= -6n \\
t_0 &= \frac{2}{m(7-p)} \ln \tan\theta \\
b &= 2(7-p) m \omega^{7-p} \sin\theta\cos\theta
\end{aligned} \tag{3.17}
$$

Also using (3.10) and (3.11) we can write $e^{2\phi}$ in (3.3) precisely in the same form as given in [16], i.e.,

$$
e^{2\phi} = F^{(3-p)/2} \left(\frac{f_-}{f_+}\right)^{pn} \tag{3.18}
$$

and finally by taking a Hodge duality on F_{8-p} given in (3.3) we find

$$
\begin{aligned}
*F_{p+2} &= -e^{(p-3)\phi/2} e^{2f(p+1)} b \, d\hat{t} \wedge dx_1 \wedge \cdots \wedge dx_{p+1} \\
&= 2(7-p) m \sin\theta\cos\theta \omega^{7-p} t^{p-8} \frac{1}{F^2} \frac{1}{f_+ f_-} \left(\frac{f_-}{f_+}\right)^n dt \wedge dx_1 \wedge \cdots \wedge dx_{p+1} \\
&= \sin\theta\cos\theta \frac{d}{dt}\left(\frac{C}{F}\right) dt \wedge dx_1 \wedge \cdots \wedge dx_{p+1}
\end{aligned} \tag{3.19}
$$

where $C = (f_-/f_+)^{(n+m)/2} - (f_-/f_+)^{(n-m)/2}$. This is the form of field strength given in [16]. In obtaining (3.19) we have used the scaling of KMP coordinates x_i, $i = 1, \ldots, p+1$ as $x_i \to (2\omega^{7-p})^{1/(p+1)} x_i$.

We now show the mapping of the CGG solution and the KMP solution for SD3-brane. As we have already mentioned that β can be eliminated in this case by just renaming the parameters α, t_0, c_1 as in (3.4) along with the coordinate rescaling $\hat{t} \to \hat{t}/\beta$ and $x_i \to x_i/\beta^{1/4}$ for $i = 1, 2, 3, 4$. Note from (3.3) that if we demand $e^{2\phi} \to 1$ as $\hat{t} \to 0$ in this case then c_2 must vanish. The SD3-brane solution of CGG then reduces to,

$$
\begin{aligned}
ds^2 &= \left(\frac{1}{\sinh 4\hat{t}}\right)^{5/2} \left(\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{1/2} \left(\frac{b}{4\alpha}\right)^{1/2} e^{c_1\hat{t}/2} \left[-d\hat{t}^2 + \sinh^2 4\hat{t} dH_5^2\right] + \\
&\quad + \left(\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0)\right)^{-1/2} \left(\frac{b}{4\alpha}\right)^{-1/2} e^{c_1\hat{t}/2} dx_{(4)}^2 \\
e^{2\phi} &= e^{2c_1\hat{t}} \\
F_5 &= \frac{b}{\sqrt{2}} (1 + *) \, \epsilon(H_5)
\end{aligned} \tag{3.20}
$$

In the above $\chi = 8$ and the parameters c_1 and α satisfy

$$
c_1^2 + 16\alpha^2 = 40 \tag{3.21}
$$

Also the five-form field strength is self-dual.

Now with the coordinate transformation (3.7) and (3.8) with $p = 3$ we get

$$
\left(\frac{1}{\sinh 4\hat{t}}\right)^{5/2} \left[-d\hat{t}^2 + \sinh^2 4\hat{t} \, dH_5^2\right] = \frac{(f_+ f_-)^{1/2}}{2^{1/2}\omega^2} \left(-dt^2 + t^2 dH_5^2\right) \tag{3.22}
$$

and we also rewrite

$$
\begin{aligned}
\cosh\frac{\chi\alpha}{2}(\hat{t}-t_0) \left(\frac{b}{4\alpha}\right) e^{c_1\hat{t}} &= 2\omega^4 \left(\frac{f_-}{f_+}\right)^n \left[\cos^2\theta \left(\frac{f_-}{f_+}\right)^{\frac{n-m}{2}} + \sin^2\theta \left(\frac{f_-}{f_+}\right)^{\frac{n+m}{2}}\right] \\
&= 2\omega^4 \left(\frac{f_-}{f_+}\right)^n F
\end{aligned} \tag{3.23}
$$

where we have defined

$$2\omega^4 \cos^2\theta = \frac{1}{2} e^{-\frac{\chi\alpha}{2} t_0} \left(\frac{b}{4\alpha}\right)$$
$$2\omega^4 \sin^2\theta = \frac{1}{2} e^{\frac{\chi\alpha}{2} t_0} \left(\frac{b}{4\alpha}\right) \tag{3.24}$$

Again from the consistency of eq. (3.23) and also from the relation (3.24) we obtain the relations between the parameters m, n, ω, θ of KMP solution and α, c_1, t_0, b of CGG solution in the same form as obtained in eq. (3.16) and eq. (3.17) with $p = 3$. With these we find that the metric and dilaton given in eq. (3.20) reduce to,

$$ds^2 = F^{-1/2} \left(\frac{f_-}{f_+}\right)^{n/2} (f_+ f_-)^{1/2} \left(-dt^2 + t^2 dH_5^2\right) + F^{-1/2} \left(\frac{f_-}{f_+}\right)^n dx_{(4)}^2$$
$$e^{2\phi} = \left(\frac{f_-}{f_+}\right)^{3n} \tag{3.25}$$

which have the same form as in [16]. In the above we have rescaled the coordinates $x_{1,2,3,4} \to (2\omega^4)^{1/4} x_{1,2,3,4}$. Finally, F_5 can be written in KMP parameters as,

$$F_5 = \frac{1}{\sqrt{2}} 8m\omega^4 \sin\theta\cos\theta (1 + *)\epsilon(H_5) \tag{3.26}$$

Thus we have shown the complete mapping of SDp-brane solutions of type II string theory obtained by CGG and KMP.

4. Conclusion

To summarize, we have shown in this paper the complete mapping of SDp-brane solutions of type II string theory obtained by Chen-Gal'tsov-Gutperle and Kruczenski-Myers-Peet. After carefully eliminating some of the parameters we have indicated that the solution of CGG is a four parameter solution and therefore is more general than the three parameter solution of KMP. In contrast to the CGG solution, KMP solution uses a specific boundary condition of the metric and the dilaton. We have shown that when the same boundary condition of the KMP solution is imposed upon the CGG solution, then one of the parameters of the CGG solution is removed and therefore both the solutions become three parameter solutions. Only under this condition both the solutions become identical and are related by a coordinate transformation as well as a Hodge duality of the field strength. The three parameters of both the solutions are related non-trivially to one another and we have explicitly given these relations. Since space-like Dp-branes, particularly the supergravity descriptions are important to understand the (time-like) holography of dS/CFT correspondence, we hope that the equivalence of two supergravity descriptions of SDp-branes shown in this paper will be helpful for this purpose as well as to understand the physical properties of these unusual branes.

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